

Calculus I
Section 5.5 – Area Approximation Methods

1. Use the table of values to approximate $\int_0^{10} f(x)dx$ using the indicated method.

x	0	2	4	6	8	10
$f(x)$	32	24	12	-4	-20	-36

- a. LRAM
- b. RRAM
- c. TRAP
- d. Simpson

2. Use the table of values to approximate $\int_0^6 f(x)dx$ using the indicated method.

x	0	1	2	3	4	5	6
$f(x)$	-6	0	8	18	30	50	80

- a. LRAM
- b. RRAM
- c. TRAP
- d. Simpson

3. The following table gives dye concentrations for a dye-concentration cardiac-output determination. Total cardiac output (in L/min) can be calculated as $\frac{336}{\int_2^{24} C(t)dt}$. Estimate the total cardiac output by approximating the definite integral using:

Seconds after injection t	Dye Concentration C
2	0
4	0.6
6	1.4
8	2.7
10	3.7
12	4.1
14	3.8
16	2.9
18	1.7
20	1.0
22	0.5
24	0

a. TRAP

b. Simpson

4. Oil is leaking out of a tanker damaged at sea. The damage to the tanker is worsening as evidenced by the increased leakage each hour. The total quantity of oil (in gallons) that escapes in 8 hours is $\int_0^8 L(t)dt$, where t is in hours and $L(t)$ is in gallons/hour. Approximate the quantity of oil that escapes using the indicated method.

t	0	1	2	3	4	5	6	7	8
$L(t)$	50	70	97	136	190	265	369	516	720

a. LRAM

b. RRAM

c. TRAP

d. Simpson

5. The table shows the velocity of a model train engine moving along a track for 10 sec. Estimate the distance traveled by the engine by approximating $\int_0^{10} v(t)dt$ using the indicated method.

a. TRAP

Time (sec)	Velocity (in/sec)
0	0
1	12
2	22
3	10
4	5
5	13
6	11
7	6
8	2
9	6
10	0

b. Simpson

6. A rectangular swimming pool is 30 ft wide and 50 ft long. The table shows the depth $h(x)$ of the water at 5-ft. intervals from one end of the pool to the other. The volume of the pool can be calculated by the expression: $1500 \int_0^{50} h(x)dx$. Estimate the volume of the pool using the indicated method (what would the units be?)

a. LRAM

Position (ft) x	Depth (ft) $h(x)$
0	6.0
5	8.2
10	9.1
15	9.9
20	10.5
25	11.0
30	11.5
35	11.9
40	12.3
45	12.7
50	13.0

b. RRAM

c. TRAP

d. Simpson

Calculus I
Section 5.5 - Riemann Sums Table

1. Use the table of values to approximate $\int_0^{10} f(x)dx$ using the indicated method.

x	0	2	4	6	8	10
$f(x)$	32	24	12	-4	-20	-36

$$\Delta x = 2$$

- a. LRAM

$$2(32 + 24 + 12 + -4 + -20) = \boxed{88}$$

- b. RRAM

$$2(24 + 12 + -4 + -20 + -36) = \boxed{-48}$$

- c. TRAP

$$\frac{1}{2}(2)(32 + 2(24) + 2(12) + 2(-4) + 2(-20) + -36) = \boxed{20}$$

2. Use the table of values to approximate $\int_0^6 f(x)dx$ using the indicated method.

x	0	1	2	3	4	5	6
$f(x)$	-6	0	8	18	30	50	80

$$\Delta x = 1$$

- a. LRAM

$$1(-6 + 0 + 8 + 18 + 30 + 50) = \boxed{100}$$

- b. RRAM

$$1(0 + 8 + 18 + 30 + 50 + 80) = \boxed{186}$$

- c. TRAP

$$\frac{1}{2}(1)(-6 + 2(0) + 2(8) + 2(18) + 2(30) + 2(50) + 80) = \boxed{143}$$

$$\Delta x = 2$$

3. The following table gives dye concentrations for a dye-concentration cardiac-output determination. Total cardiac output (in L/min) can be calculated as $\frac{336}{\int_2^{24} C(t)dt}$. Estimate the total cardiac output by approximating the definite integral using:

Seconds after injection t	Dye Concentration C
2	0
4	0.6
6	1.4
8	2.7
10	3.7
12	4.1
14	3.8
16	2.9
18	1.7
20	1.0
22	0.5
24	0

a. TRAP

$$\frac{1}{2}(2)(0+2(0.6)+2(1.4)+2(2.7)+2(3.7)+2(4.1)+2(3.8)+2(2.9)+2(1.7)+2(1.0)+2(0.5)+0) = 44.8$$

$$\Rightarrow \boxed{336/44.8 = 7.5 \text{ L/min}}$$

b. LRAM

$$2(0+0.6+1.4+2.7+3.7+4.1+3.8+2.9+1.7+1.0+0.5) = 44.8$$

$$\boxed{\frac{336}{44.8} = 7.5 \text{ L/min}}$$

4. Oil is leaking out of a tanker damaged at sea. The damage to the tanker is worsening as evidenced by the increased leakage each hour. The total quantity of oil (in gallons) that escapes in 8 hours is $\int_0^8 L(t)dt$, where t is in hours and $L(t)$ is in gallons/hour. Approximate the quantity of oil that escapes using the indicated method.

$$\Delta x = 1$$

t	0	1	2	3	4	5	6	7	8
$L(t)$	50	70	97	136	190	265	369	516	720

a. LRAM $1(50+70+97+136+190+265+369+516) = \boxed{1693}$

b. RRAM $1(70+97+136+190+265+369+516+720) = \boxed{2363}$

c. TRAP $\frac{1}{2}(1)(50+2(70)+2(97)+2(136)+2(190)+2(265)+2(369)+2(516)+720) = \boxed{2028}$

5. The table shows the velocity of a model train engine moving along a track for 10 sec. Estimate the distance traveled by the engine by approximating $\int_0^{10} v(t)dt$ using the indicated method.

a. TRAP

$$\Delta x = 1$$

$$\frac{1}{2} (1)(0 + 2(12) + 2(22) + 2(10) + 2(5) + 2(13) + 2(11) + 2(6) + 2(2) + 2(6) + 0) = \boxed{87}$$

b. RRAM

$$1 (12 + 22 + 10 + 5 + 13 + 11 + 6 + 2 + 6 + 0) = \boxed{97}$$

Time (sec)	Velocity (in/sec)
0	0
1	12
2	22
3	10
4	5
5	13
6	11
7	6
8	2
9	6
10	0

6. A rectangular swimming pool is 30 ft wide and 50 ft long. The table shows the depth $h(x)$ of the water at 5-ft. intervals from one end of the pool to the other. The volume of the pool can be calculated by the expression: $30 \int_0^{50} h(x)dx$. Estimate the volume of the pool using the indicated method (what would the units be?) $\Delta x = 5$

a. LRAM $30 \left[5 (6 + 8.2 + 9.1 + 9.9 + 10.5 + 11.0 + 11.5 + 11.9 + 12.3 + 12.7) \right]$

$$= \boxed{15,465}$$

b. RRAM

$$30 \left[5 (8.2 + 9.1 + 9.9 + 10.5 + 11.0 + 11.5 + 11.9 + 12.3 + 12.7 + 13.0) \right] = \boxed{16,515}$$

c. TRAP

$$30 \cdot \frac{1}{2} \cdot 5 \left[6 + 2(8.2) + 2(9.1) + 2(9.9) + 2(10.5) + 2(11.0) + 2(11.5) + 2(11.9) + 2(12.3) + 2(12.7) + 13 \right]$$

$$= \boxed{15,990}$$

Position (ft) x	Depth (ft) $h(x)$
0	6.0
5	8.2
10	9.1
15	9.9
20	10.5
25	11.0
30	11.5
35	11.9
40	12.3
45	12.7
50	13.0

NOTE: For the following two problems, the widths of the subintervals are not all equal.

7. Use the table of values to approximate $\int_0^{10} f(x)dx$ using the indicated method.

	2		1	3	3	1
x	0	2	3	6	9	10
f(x)	32	24	12	-4	-20	-36

- a. LRAM

$$2(32) + 1(24) + 3(12) + 3(-4) + 1(-20) = \boxed{92}$$

- b. RRAM

$$2(24) + 1(12) + 3(-4) + 3(-20) + 1(-36) = \boxed{-48}$$

- c. TRAP

$$\frac{1}{2}(2)(32+24) + \frac{1}{2}(1)(24+12) + \frac{1}{2}(3)(12+(-4)) + \frac{1}{2}(3)(-4+(-20)) + \frac{1}{2}(1)(-20+(-36)) = \boxed{22}$$

8. Use the table of values to approximate $\int_0^{14} f(x)dx$ using the indicated method.

	2		1	3	2	2	4
x	0	2	3	6	8	10	14
f(x)	-6	0	8	18	30	50	80

- a. LRAM

$$2(-6) + 1(0) + 3(8) + 2(18) + 2(30) + 4(50) = \boxed{308}$$

- b. RRAM

$$2(0) + 1(8) + 3(18) + 2(30) + 2(50) + 4(80) = \boxed{542}$$

- c. TRAP

$$\frac{1}{2}(2)(-6+0) + \frac{1}{2}(1)(0+8) + \frac{1}{2}(3)(8+18) + \frac{1}{2}(2)(18+30) + \frac{1}{2}(2)(30+50) + \frac{1}{2}(4)(50+80) = \boxed{425}$$